# Lecture 1

Alert you to opportunities for better writing.

more clear, more precise, more concise, more forceful

Refs: Vivaldi's book and Halmos' paper (both on website)

# How to write mathematics?

There is no recipe, yet it helps to learn tricks of the trade.

1. Have something to say.
2. Have someone to say it to.
3. Organize what you want to say.
4. Write it, rewrite it, re-rewrite it,...
5. Work hard on details.

Today: focus on improving (English) writing segments.

# Grammar quick rules

- **Complete sentences** (subject, verb, punctuation...)
  **Bad**: *"A cubic polynomial."*
  **Good**: *"Consider the cubic polynomial* $p(x)$*."*

- **Subject–verb agreement**
  **Bad**: *"The set of primes are infinite."*
  **Good**: *"The set of primes is infinite."*

- **Pronoun–antecedent agreement**
  **Bad**: *"Each function is greater than their minimum."*
  **Good**: *"Each function is greater than its minimum."*

- **Only judiciously split infinitives**
  **Bad**: *"We have to thoroughly examine this proof."*
  **Good**: *"We have to examine this proof thoroughly."*

# Common misuses

- **its** (possessive) vs. **it's** (it is)
  **Bad**: *"It's eigenvalues are real."*
  **Good**: *"Its eigenvalues are real."*

- **that** (restrictive) vs. **which** (non-restrictive)
  **Good**: *"The lemma that we use is based on . . ."*
  **Good**: *"Lemma 3, which we will use to prove Theorem 1, is based on . . ."*

- **fewer** (countable) vs. **less** (uncountable)
  **Bad**: *"less primes"*, *"fewer area"*.
  **Good**: *"fewer primes"*, *"less area"*.

# Common misuses, cont.

- **principle** (noun) vs. **principal** (adjective)
  **Bad**: *"the principal of induction"; "a principle bundle."*
  **Good**: *"the principle of induction"; "a principal bundle."*

- **where** as relative adverb means "in which", not "of which"
  **Bad**: *"Consider the logarithmic function, where its derivative is positive."*
  **Good**: *"Consider the logarithmic function, whose derivative is positive."*

- **when** as an adverb is not suitable for definitions
  **Bad**: *"A prime number is when there are no proper divisors."*
  **Good**: *"A prime number is an integer with no proper divisors."*

# Numbers and symbols

- Sentences with symbols should still be correct sentences.
  **Bad**: *"$a < b$ $a \neq 0$."*
  **Good**: *"We have $a < b$ and $a \neq 0$."*
  *Or: "We find that $a < b$ and $a \neq 0$."*
  *Or: "Let $a < b$, with $a \neq 0$."*

- Avoid unnecessary symbols.
  **Bad**: *"Every differentiable real function $f$ is continuous."*
  **Good**: *"Every differentiable real function is continuous."*

- Prefer not to begin a sentence with a symbol.
  **Bad**: *"$\rho$ is a rational number with odd denominator."*
  **Good**: *"The number $\rho$ is rational with odd denominator."*

# Numbers and symbols, cont.

- Spell out small counting numbers.
  **Bad**: *"The equation has 4 solutions."*
  **Good**: *"The equation has four solutions."*
  **Good**: *"The equation has 127 solutions."*

- Use numerals for specific numbers.
  **Bad**: *"Both three and five are prime numbers."*
  **Good**: *"Both 3 and 5 are prime numbers."*

- Do not combine operators with words.
  **Bad**: *"The difference* $b - a$ *is* $< 0$*."*
  **Good**: *"The difference* $b - a$ *is negative."*

# Numbers and symbols, cont.

- Separate adjacent formulas or symbols with words.
  **Bad**: *"Consider* $A_n$, $n < 5$.
  **Good**: *"Consider* $A_n$, *where* $n < 5$."
  *For displayed equations, can use space to allow adjacent formulas.*

- **Implication symbol** $\Rightarrow$ to be used only inside formulas.
  **Bad**: *"*$a$ *is an integer* $\Rightarrow$ $a$ *is rational."*
  **Good**: *If* $a$ *is an integer, then* $a$ *is rational.*

- **Therefore symbol** $\therefore$ to be avoided altogether.
  **Bad**: *"*$\therefore x = 3$."
  **Good**: *"Hence, we have* $x = 3$."
  **Good**: *"... and therefore* $x = 3$."

# Style: clarity beats cleverness

- **Prefer short, direct sentences.**
  **Bad**: *"We note the fact that the polynomial* $2x^2 - x - 1$ *has the coefficient of the* $x^2$ *term positive."*
  **Good**: *"The leading coefficient of the polynomial* $2x^2 - x - 1$ *is positive."*

- **Avoid vague intensifiers.**
  **Bad**: *"The proof is very easy, as it makes an elementary use of the triangle inequality."*
  **Good**: *"The proof uses the triangle inequality."*

# Style, cont.

- **Prefer active voice.**
  **Bad**: *"The convergence of the above series will now be established."*
  **Good**: *"We now establish the convergence of the above series."*

- **Emphasize the topic.**
  **Bad**: *"An example of a transcendental function is the logarithm."*
  **Good**: *"The logarithm is an important example of a transcendental function."*

# Style, cont.

- Make a choice of expository voice: *"we"* vs. *"I"* vs. *"one"*.
- Use foreign expressions sparingly.
- Avoid distraction through obtrusive expressions.
- Avoid overuse of *"hence"* and *"therefore"*.

  *Alternatives: "thus", "so", "it follows that", "as a result", "consequently".*
- Avoid overuse of *"get"*, *"very"*, *"nice"*.

  *Alternatives: "obtain", "significantly", "pleasant", etc.*
- **Take the reader into your confidence.**
- **Ripen your text by letting it sit for a while.**

# What's fishy?

1. *If $R$ is a commutative semisimple ring with unit and if $x$ and $y$ are in $R$, then $x^2 - y^2 = (x - y)(x + y)$.*

   What does this have to do with semisimplicity or with having a unit?

2. *Every complex number is the product of a non-negative number and a number of absolute value 1.*

   Isn't it misleading not to alert the reader to the special case of 0?

3. *Theorem: Without loss of generality, assume that the function $f$ satisfies $f(0) = 0$. ...*

   "Without loss of generality" belongs in a proof, not in a theorem statement.

Statements may be correct, yet still confuse, mislead or be out of place.

# Preparation and structure

- Draft first; revise often. Read aloud for flow and clarity.
- Know your audience; calibrate background and pace.
- One idea per paragraph; signpost transitions between ideas.
- Strong opening and closing: motivate, then land the plane.
- Mathematics and language have to be correct.

# Exercises 1.1 from Vivaldi's book

1. **Bad**: $a$ is positive.
   **Good**: The number $a$ is positive.

2. **Bad**: Two is the only even prime.
   **Good**: The only even prime is $\mathbf{2}$.
   *Alternatively: "The integer $\mathbf{2}$ is the only even prime."*

3. **Bad**: If $x > \mathbf{0}$ $g(x) \neq \mathbf{0}$.
   **Good**: If $x > \mathbf{0}$, then $g(x) \neq \mathbf{0}$.

4. **Bad**: We minus the equation.
   **Good**: We change sign to both sides of the equation.
   *Alternatively: "We multiply both sides of the equation by $-\mathbf{1}$."*

# Exercises 1.1 from Vivaldi's book, cont.

5. **Bad**: $x^2 + 1$ has no real solution.
   **Good**: The polynomial $x^2 + 1$ has no real roots.
   *Alternatively: "The equation $x^2 + 1 = 0$ has no real solutions."*

6. **Bad**: When you times it by negative $x$, $\leq$ becomes $\geq$.
   **Good**: Multiplying both sides by a negative value of $x$, the inequality is reversed.

7. **Bad**: The set of solutions are all odd.
   **Good**: The solution set consists of odd integers.
   *Alternatively: "All solutions are odd."*

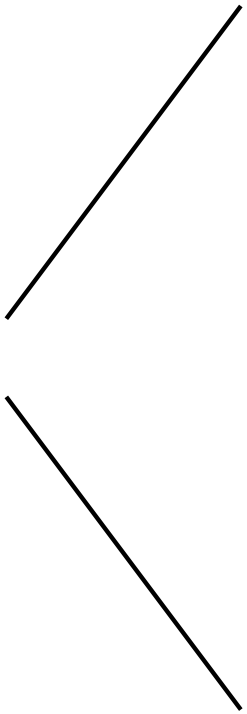
# Exercises 1.1 from Vivaldi's book, cont.

8. **Bad**: $\sin(\pi x) = 0 \Rightarrow x$ is integer.
   **Good**: If $\sin(\pi x) = 0$, then $x$ is an integer.
   *Alternatively: "The condition $\sin(\pi x) = 0$ implies that $x$ is an integer."*

9. **Bad**: An invertible matrix is when the determinant is non-zero.
   **Good**: A matrix is invertible if and only if its determinant is non-zero.

10. **Bad**: The infinete sequence has less negative terms.
    **Good**: This infinite sequence has fewer negative terms.

# Homework for 24/Sept – not to turn in



**Option A:**

Go through about half of the numbered parts of Exercises 1.2, 2.3 and 2.4 of Vivaldi's book.

**Option B:**

Identify twelve writing problems (spelling, grammar, typography, bad notation, unclear statements, etc) in the paper by Erdös and Szemerédi available on the webpage.

# Anticipate homework for 8/Oct

Choose an elementary **definition or theorem** out of the list on the webpage for **Paper 1**.

Think about what contents you would like to see as a Math *Ersti*.

You may look at any references or tools for inspiration.

Keep in mind the given guidelines for Paper 1.

Make a sketch for your Paper 1.

LaTeX will be introduced in the next lecture.