Lecture 9

Better Writing

For an Audience of Mathematicians

Ref: chapter 6 of Vivaldi's book

Slides prepared with assistance from ChatGPT 5.

Recall lecture 1: How to write mathematics?

☞ There is no recipe, yet it helps to learn tricks of the trade.

1. Have something to say.
2. Have someone to say it to.
3. Organize what you want to say.
4. Write it, rewrite it, re-rewrite it,...
5. Work hard on details.

Today: focus on improving mathematical writing.

- Choosing vocabulary and improving formulas
- Theorems, proofs and rhetorical devices
- Learn by doing and by reading

Choosing words and symbols

- Wisely choose your **primitive** words and symbols.

  Those that you use without an explanation of their meaning.

  Exs: "*eigenvalue*", "*Euler's number*", "*exterior product*"

- Watch out for terms whose meaning depends on context.

  These you should clarify.

  Exs: "*exceptional set*", "*regularity*", "*cluster*"

- Define words and symbols **the first time you use them**.

  Do not rename them later.

- Choose symbols wisely. **Bad**: *"Let n be a real number."*

  Do not introduce unnecessary symbols.

Name things once; keep names consistent

Bad: *"Consider the function f. Now take the map g defined by $g = f$..."*

Good: *"Consider the function f. We will keep this notation throughout."*

Depending on context, stick to your option:

- **function** vs. **map** vs. **transformation**
- **variable** vs. **argument** vs. **unkonwn** vs. **indeterminate**
- **element** vs. **member** vs. **point** vs. **term**
- **component** vs. **coordinate** vs. **entry**

Be accurate with words

- Example: Distinguish **equation**, **inequality**, and **identity.**

  Bad: *"The equation* $x^2 - 3 \geq 0$*..."*
  Good: *"The inequality* $x^2 - 3 \geq 0$*..."*

  Bad: *"The equation* $x^2 - 1 = (x-1)(x+1)$*..."*
  Good: *"The identity* $x^2 - 1 = (x-1)(x+1)$*..."*

  Bad: *"The identity* $x = \sqrt{x^2}$*..."*
  Good: *"The equation* $x = \sqrt{x^2}$*..."*

- "**Any**" can be ambiguous, as it depends on the context:

  Bad "For any $\varepsilon > 0$ there exists $\delta > 0$ such that..."
  Good "For every $\varepsilon > 0$ there exists $\delta > 0$ such that..."

Improving formulas

Bad: $f(x) = \dfrac{14x - 2x^3 - 2x^2 + 14}{-2x - 4}$ [untidy formula]

Good: $f(x) = \dfrac{x^3 + x^2 - 7x - 7}{x + 2}$ [if the degree or coefficients matter]

Good: $f(x) = \dfrac{(x + 1)(x^2 - 7)}{x + 2}$ [to solve $f(x) = 0$]

Good: $f(x) = x^2 - x - 5 + \dfrac{3}{x + 2}$ [for integration]

Smaller formulas may be included inline.

Use displaymode when a line-break increases legibility.

Punctuate equations as part of sentences.

Theorems and proofs

Theorem framing: give hypotheses first, then a crisp claim.

Bad: **Theorem.** *Let G be a group. Then the result holds if G is finite and abelian.*
Good: **Theorem.** *Let G be a finite abelian group. Then G is a direct product of cyclic groups of prime power order.*

Start with hypotheses: structural (G is a group) and specific (finite, abelian).

Proof framing: say what you'll do; say when you're done.

Bad: **Proof.** *We argue as before.* [too vague]
Good: **Proof.** *We proceed by induction on n. The base case $n = 1$ is immediate. For the induction step, assume... This completes the induction and the proof.* □

Organize longer arguments with the use of lemmas.

Counterexamples and rhetorical devices

Bad: "This reciprocal claim is obviously false." [no evidence]

Good: "Continuity does not imply differentiability: take $f(x) = |x|$, which is continuous but not differentiable at 0."

Prefer a minimal counterexample.

Bad: "A vector field is like wind." [too vague metaphor]

Good: "Think of a vector field as assigning a velocity at each point, as wind does to a leaf; integral curves are the leaf's trajectories."

Keep the correspondence between parts of the analogy and the math explicit.

Bad: "We compute a determinant." [no motivation]

Good: "What changes if we swap two rows? Since the determinant is alternating, it flips sign; hence,..."

A well-placed question previews the key property you will use.

What's fishy?

Bad: "Every linear map is diagonalizable."

False / missing hypotheses.

Good: "Every linear map on a finite-dimensional vector space with distinct eigenvalues is diagonalizable."

Bad: "$A \subseteq B$ therefore $|A| \leq |B|$ holds."

Symbols glued to prose.

Good: "Since $A \subseteq B$, we have $|A| \leq |B|$."

Bad: "A function is Lipschitz if $|f(x) - f(y)| \leq L|x - y|$."

Imprecise definition.

Good: "A function $f : X \to \mathbb{R}$ is *Lipschitz with constant* $L \geq 0$ if $|f(x) - f(y)| \leq L\,|x - y|$ for all $x, y \in X$."

Recall lecture 1: Preparation and structure

- Draft first; revise often. Read aloud for flow and clarity.
- Know your audience; calibrate background and pace.
- One idea per paragraph; signpost transitions between ideas.
- Strong opening and closing: motivate, then land the plane.
- Mathematics and language have to be correct.

Further tips complementing those of Lecture 1

- First draft a skeleton.

  Paper sections, raw results/conjectures, key ideas,...

- Fill in the material. Be patient with the revisions!

  The first sentence of each paragraph should be key.

- Write the **introduction** last.
  The introduction is arguably the most important part.

  – *Why would the reader want to read your paper?*
  – *What's the main result/content of your paper?*

- Point to your references.

The real thing?

What's good mathematical writing?

...universal themes ...enduring impact ...aesthetic appeal

What's better mathematical writing?

more clear, more precise, more concise, more forceful

☞ **Read good papers**

Example: two-page paper by Whitney

An even function $f(x) = f(-x)$ (defined in a neighborhood of the origin) can be expressed as a function $g(x^2)$; $g(u)$ is determined for $u \geq 0$, but not for $u < 0$. We wish to show that g may be defined for $u < 0$ also, so that it has roughly half as many derivatives as f. A similar result for odd functions is given.

opening paragraph of
paper by H. Whitney in Duke Math. J. 10 (1943), 159–160.

Small paper with big impact

DIFFERENTIABLE EVEN FUNCTIONS

By Hassler Whitney

An even function $f(x) = f(-x)$ (defined in a neighborhood of the origin) can be expressed as a function $g(x^2)$; $g(u)$ is determined for $u \geq 0$, but not for $u < 0$. We wish to show that g may be defined for $u < 0$ also, so that it has roughly half as many derivatives as f. A similar result for odd functions is given.

Theorem 1. *An even function $f(x)$ may be written as $g(x^2)$. If f is analytic, of class C^∞ or of class C^{2s}, g may be made analytic, of class C^∞ or of class C^s, respectively.*

If $f(x) = \sum a_i x^i$ is even and analytic, then $a_i = 0$ for i odd, and we may set $g(u) = \sum a_{2i} u^i$, which is analytic.

Suppose f is even and of class C^{2s}. Then Taylor's formula gives

$$f(x) = a_0 + a_1 x^2 + \cdots + a_{s-1} x^{2s-2} + x^{2s}\phi(x). \tag{1}$$

By Theorem 1 of [3], ϕ is even and continuous, and of class C^{2s} for $x \neq 0$, and

Homework for 19/Nov: Paper 6

Paper 6 = Your revised version of your Paper 4
Check guidelines for Paper 6 on course webpage.

1. Tomorrow (13/Nov) find on Moodle report(s) on your Paper 4.
2. Revise your original Paper 4 based on that and your own updates.
3. Respond to the report(s) in up to one page in LaTeX.
4. Upload **four files** by 19/Nov 22:00 – see guidelines.
5. NEW Indicate your topic in a **submission comment**.

- Remember to write name and title on your paper.
- Remember target audience: colleagues here.
- Check your references, especially when from AI.

Homework for 26/Nov: Paper 7

Paper 7 = Template to share on Overleaf
Check guidelines for Paper 7 on course webpage.

- Choose a template type.
  Slides, or poster, or lecture notes, or cheat sheet, or exercise sheets, or CV.
- You may start from a template found online, e.g. on Overleaf.
  Do not ask AI to provide the starting template.
- Get acquainted with it and fill it out for illustration.
  Simply slightly adjust it to your taste.
- **NEW** Get bonus by shortly presenting template on 10/Dec.

Paper 7: if you do slides

Slides using LaTeX: `\documentclass{beamer}`

-  Some LaTeX commands need to be adjusted or protected.
- It can help to watch a tutorial:

  https://www.overleaf.com/learn/latex/Beamer_Presentations%3A_A_Tutorial_for_Beginners_(Part_1)

- Two or three slides/pages called "`frames`" in `beamer` are enough.
- You may recycle material from Paper 6 or from Paper 3, but...
- Don't simply cut-and-paste from paper $\longrightarrow$ **avoid overload!**