“How to write mathematics badly” by Jean–Pierre Serre

Transcribed by Maxine Calle

Transcriber’s note: In Fall 2003, Jean–Pierre Serre gave a public lecture on writing mathematics as part of Harvard’s Basic Notions Seminar [1]. I think this is a great lecture, and since I do not always have the time watch the video, I thought it would be helpful to transcribe it. I hope this transcription will also allow more people to enjoy his wisdom. I have tried to remain true to the audio (from [2]), although I have taken the liberty of omitting some of the filler words and repetitions. In places where I noted aspects of the video other than his speech, I have used square brackets and italics, [*like this*]. I have also separated the lecture into sections with time markers (again, referencing [2]). Unfortunately there are some parts where I cannot make out what he is saying; I have indicated these areas with round parentheses and question marks, (like this, if I have some idea?) or just (?) if I don’t know.

I am grateful to Uros Colovic, Alex Dumortier, Arthur Langlard, and Jean-Pierre Serre himself for feedback and corrections that have improved the quality of the transcript. Comments and corrections are welcomed and should be directed to maxinecalle@gmail.com. The most recently updated version is available on my website: `http://web.sas.upenn.edu/callem/`.

Introduction (0:10)

Maybe I should explain before I really start that I was asked to speak on writing mathematics, and probably with the idea that I could explain how to write mathematics well. So I told (Taylor?) that that would be difficult and presumptuous too. But on the other hand, I feel I am an expert in how to write mathematics badly. So what I want to describe today is how you write mathematics badly. Maybe by taking the opposite we might manage to do better.

On Theorems (0:48)

So let's imagine somebody who really wants to get rid of the theorem he has proved. He wants to publish it — he or she, the sexes (are) indifferent in this — he wants to publish it as soon as possible. He doesn't care that he's not read or understood. And he might prefer maybe not to be understood sometimes. So what does he do? Well, first he has to choose a title that doesn't give any information at all. For instance, he can say "A proof of a theorem of" and then either take somebody absolutely unknown [*writes "so-and-so"*], so that we don't know who that is, or else say "A proof of a theorem of Euler." The two things are essentially equivalent; they give no information. Then he has to start by stating his theorem. Well, he will call it his main theorem. Nowadays, you see, theorems are indexed by things like $2, 3, 4$, and so on, so nobody will check there is only one theorem in the paper. He can still call it the main theorem. What notation does he take? He takes the notation which are in his head. And he believes that every reader will know his notation. He will say my main theorem is

Theorem. *An fpph ATT is regular.*

He believes that if you don't know what ATT and fpph — of course, he made a misprint. He wanted to write "fppf," but putting a misprint in a theorem makes it even harder to understand. As for regular, he has a good point, regular is well-defined in mathematics. So well-defined, there are about two dozen different definitions. So, that's okay, he starts with that.

On References (3:13)

He might be afraid of the referee. The referee might ask him for a reference for ATT and maybe also for regular. In that case, (?) wait for at least one or two pages to give reference for whatever ATT or fppf. And the reference is careful (?) I mean when you give a reference and you don't want the reference to be checkable, what you do is you do the following: give a reference to either to a long book, where 600 pages is a good size, without any more reference, without pages. Well, even if you write on the theorem of Euler, you can refer to the complete works of Euler. They have not been published yet entirely. But if you refer to SGA or EGA, you have a good chance also. But, speaking of references, there is something which you cannot beat, which is better than this. Refer to, for the definition of ATT, "cf. [H]." And then you look in the bibliography, and you find H is [*writes "D. Hilbert, private communication"*]. But at the moment, bibliographies are really full of references of that kind which cannot be checked by anybody. Well okay, the name of Hilbert may be a little showy but it is exactly the same, it is so-and-so. It should be in the bibliography.

On Proofs (5:15)

So this is the way you start: you start with not explaining your notations, and when you do explain them, making impossible references. After that, you have to start giving the proof. But there are many ways of making a proof difficult to follow. [*Writes "Typography."*] There are some typographical things that you can do to make life a bit more difficult. For example, "We have $f(x) = \sin(x) \cdot \cos(x). \cos(x)$ being continuous..." and so on. You have a dot which is a normal role of a dot [*circles dot between the* $\cos(x)$ *and* $\cos(x)$] which separating things and you have also that dot [*indicates dot between* $\sin(x)$ *and* $\cos(x)$]. So really, it's nice and confusing enough. Very easy to arrange because we could always insert "the function" [*writes "the function" before* $\cos(x)$]. If you want, you can put in your mind never to begin a sentence with a symbol. I used to have a typewriter which would refuse to do that. When I was typing something, if I typed something starting with f, I heard it screaming. It is very easy to put this in your mind. That is one thing you can do. What else can you do to make life complicated? Well, life complicated and also make life easier for you — complicated for the reader, and simpler for you. Bad writing is really something which is easy, as I said, for the writer and the real job is left for the reader. How do you manage that? Which is very close to what I will call cheating, it's not really cheating, in the sense that the mathematician normally doesn't want to cheat. But he wants to make life easy. So what can you do? Well, (there are many things?). One of them is rather serious. It is the following: you write your proof in the (?). You write "Lemma 1" or 1-point-something, with a different system, and then something rather easy. And you give a full proof. And then you do Lemma 2, same style. Then you get to Lemma 3, and here you have put all the difficulties of the theorem, all the computations, they are all in Lemma 3. Lemma 3, you either say, if you're mildly honest, "It is a computation." But there are variants of that. You could always say "It follows from the definitions," and this cannot be beaten. Or any kind of thing, but there is one thing that I really recommend if you want to make life really hard for the reader, which is to state your Lemma 3, and then you do just that [*writes "$\Box$"*]. This was invented, I mean, I didn't know this symbol when I was a graduate student, but it was invented about twenty years later. It's extremely convenient for cheating, because when you put it...in the old days, you had to write something like this [*underlines "It is a computation" and "Follows from the definitions"*] or at least give some idea why you were not putting the proof. Nowadays, you just type this beautiful symbol and nobody can ask (you this?). So that's (word?) cheating.

On Grammar (10:00)

This is just (on the style?) so to speak, but you have also (more serious, I think?) See on using grammar, English grammar, in a not totally honest way. Typical things: Theorem. And this is very frequent in papers you can find everywhere.

Theorem 1. *There exists an isomorphism* $f\colon X \to Y$*, with this and that.*

So that will be the theorem, and you do prove there is an isomorphism, or you make some kind of proof. And immediately after that you say

Theorem 2. f *is continuous*

or whatever. Now what's wrong with that? You see it is a fact that you're playing essentially with *an* isomorphism [*writes "a" above Theorem 1*], so an indefinite article. Here [*indicates Theorem 2*], if you want to write it correctly, because I started with a symbol — my typewriter will scream — so I should have said "The map." If you pretend to be precise, you say "The map of Theorem 1," which is absolutely meaningless! I mean, you find that in many many papers, (and it doesn't?) mean anything because the first theorem says that the set of isomorphisms is non-empty. This is absolutely equivalent [*writes "*$\iff \mathrm{Isom}(X, Y) \neq \emptyset$*" next to Theorem 1*]. And then you pretend that you have an f, that is, that you have chosen something there. Now most of the time, it is not entirely cheating in the sense that the proof of the theorem, sometimes, really gives you an f. And that's the one, but then you should say the map of the proof of Theorem 1. But quite often you don't even have that. But because you have switched from the indefinite article [*writes "a" next to Theorem 1*] to the definite one [*writes "the" next to Theorem 2*], you are able to say things that you have really not proven at all. And even sometimes you can claim things that are really meaningless. You're going to say, for instance, "The map f of Theorem 1 coincides with the one defined by" — well, let's choose a name at random — "Euler…" and so on. Of course, if we have not defined [*pointing at "a" and "the"*]. Well this has occurred in many (places?) in the theory. Sometimes, you can do something about that. That is, you can actually prove there is an isomorphism but you don't get a precise one. Or sometimes, like in Langland's theory, he wants to get a bijection between representations of some type and some other, and I remember lectures where people were saying that "There exists a bijection" [*throws up hands in frustration*]. Okay, I will tell them, you mean these two sets have the same cardinal. Of course, they didn't mean that at all. But at the time, they were not able to tell exactly what (?) was a bijection. But at least you should be aware, that when you state something like this [*indicates Theorem 1*], you are saying nothing more than this [*boxes "*$\Longleftrightarrow \mathrm{Isom}(X, Y) \neq \emptyset$*"*]. A worst case, that's something also...I mean, every day I receive preprints where this type of thing occurs.

Theorem 1. (a) *There exists* f*,*
(b) f *is… something.*

Well, if it is on the blackboard, I usually interrupt the speaker, I like to do that, and ask him "Do you mean that you can choose f in (a) to have property (b), or do you mean that every f having property (a) has property (b)?" Most of the time, they play safe and they say "Oh, I mean one can prove f has got property (a) and (b)." They chicken out. This is serious and actually not easy to (arrange?) instead of speaking about bad writing, I try to say something on good writing, this is not a trivial point. Because usually you make a construction, you get an f, and you want to state the properties of f. And you are in a hurry, so you state immediately theorems "There exists an f." After that, you will be (close?). What is better is to construct f first, if possible, then state "f is an isomorphism" of whatever property you have in mind, and that's okay. But that needs a little patience. Of course, you could just announce it. You could make in the commentaries, or in the introduction, you could say "We construct in paragraph 2, an f having this property." But it is better to construct the animal first, and then give its properties.

On Spelling (16:33)

What else can one do? (?) Various other things one can do...Let's make life a bit more amusing by...you can always do this: spicing things up with creative spelling. (?) called just mispelling. As a good example of misspelling, I think the best one is really this one [*writes "mispelling"*]. This is a very good example of misspelling which — it is a self-describing word, I mean, it's perfect — which comes from the idea that it is half-spelling, which it is not, no. I don't want to give you bad examples, so let's try to write it correctly. So, some misspellings, some of them are just entertaining. How often do you find "it's" as an abbreviation of sorts for "its." I have noticed that it is very often non-English or non-American people who do that mistake. When you are a foreigner and you learn English, you have a tendency to learn first the bad habits of English, and putting things like that [*indicates the apostrophe in "it's" on the board*] is not such a very good idea. Here it is used (wrongly?). But my favorite misspelling is not that one. It occurs in topology…"principle bundle." Pronunciation is perfect, there is no problem, and also a dictionary will tell you that this makes sense. And in a sense, it's a rather poetic thing. It means a principle which varies with a parameter, so that like a Möbius band: you see, when you start with a very strong principle, you make a loop, and you arrive with the opposite principle. Don't believe it is rare. I looked in the famous Google before I came. Google gives 400 examples of that [*points at "principle bundle"*]. To be true, Google gives also about 4,000 correct principal bundles. But maybe I should still write the correct "principal bundle." Also one of them which gives "principle bundle" explains that that's wrong. He tells the guy — because this is a conversation — he tells the guy "Remember, 'principle bundle' written that way…" [*writes "Has a moral fiber"*]. I didn't do it; I'm just copying what I found. I just copied. But unfortunately, in the examples given by Google, some of them were books! Where the thing was defined that way [*indicates "principle bundle"*]. And where they complained (?).

On Abbreviations (21:06)

Oh, there are many ways of writing things badly. The use of abbreviations, also, can make life complicated. I remember one for which Kodaira was not very happy.

An ell. curve with. c.m.

An elliptic curve with c.m. With c.m., and that had happened in Princeton, and I must admit I was the one writing this on the blackboard. And Kodaira told me, "What do you mean, with complex multiplication," I said "No! it's not with complex multiplication! I put a dot, you see. [*Points to dot at end of "with."*] So the dot means it's an abbreviation." And of course an abbreviation can be only that [*writes "Without"*]. So, I remembered not to do it again. Except here. So be careful about abbreviations. People have a tendency to write everything with symbols, like whatever they are [*writes* "$\forall, \exists$"], used in fact in a way not compatible with the (original?). It doesn't take very long for replacing them by "for all" and "exists."

On Commas (27:36)

Another thing which makes life difficult for the reader is the use of commas. Very often you find a statement of the following form: "If A" — and A being a complicated expression, A being like, I don't know...

If A, B, C.

B and C complicated expressions. And then the commas, you see, are used by the writer in two different meanings. It means here to (spell it?) completely, "If A and B are true, so is C." So that is, this comma [*indicates comma between A and B*] is an "and," and this one [*indicates comma between B and C*] is "hence." You have to be careful, I mean, it is really bad idea to make commas do work they are not paid to do. They should not do that. So if you want to say this [*points to "If A and B are true, so is C"*], because there is no way — it might mean "If A is true, then B and C are true." This is a standard construction.

On Verbs (24:03)

Do I have a complete list? Certainly not. Of various things one can do. Well, sometimes symbols are used to replaced verbs. That's also very disagreeable. I remember a paper in *Inventiones*, at the time I was an editor, where practically the main theorem was something like this: Theorem... No word in the theorem. The theorem was a formula. And the formula was a complicated expression... Some value of an L-function with some characters and whatnot... equals another equally complicated expression, but different, something with powers of $2i\pi$, and things..., and then this [*writes* "$\in \mathbb{Q}$"].

Theorem. *Compl. exp.* $L(\ ,x) =$ *eq. compl. expr.* $(2i\pi)^l \in \mathbb{Q}$.

Fortunately, I looked at the paper and understood that what he meant was the following: He meant that this complicated expression [*indicates left side of equality*] equals this one [*indicates right side of equality*]. This he knew already. This had been proved in the paper before. So the statement was entirely here [*circles "* $\in$*" and draws arrow to it*]. He meant that some number is a rational number. But you had to see, in the long list of symbols, which one was a verb! So please don't use, if possible, symbols for verbs.

On Different Types of "Cheating" (25:50)

Okay, so this is about more or less the grammar of writing, and what you can do with the grammar. There are more serious, or different types of. . . I hesitate to say "Cheating," because that is a bit strong. When you speak in a foreign language, you have a tendency to use the strong words, not the weak words. So you have to excuse me for that. Different types of...well, cheating. Now this, this (?) differently in the different branches of mathematics. So one has to look at different branches.

Homological Algebra

For instance, homological algebra is (?). Well, here the usual cheating is that you claim all the diagrams are commutative. Which is sometimes the case. And if there are two people have constructed two arrows with the same origin and the same extremities, they are equal. Two "naturally defined arrows" are equal. So for instance, you prove an isomorphism between this (cohomology?) group and that one, and you recover the theorem of, say, Eilenberg-MacLane. I remember telling that to Eilenberg and him telling me "Well, Serre, you have not proved it's the same map. You don't know. Might be the opposite." So, to prove actually the diagram is commutative, this is a serious question, it is not really cheating — there is no very good way in mathematics how to prove that the diagram is commutative. Theoretically, you could say, if it is a very simple diagram like that

$$\begin{array}{ccc} \bullet & \longrightarrow & \bullet \\ \downarrow & & \downarrow \\ \bullet & \longrightarrow & \bullet \end{array}$$

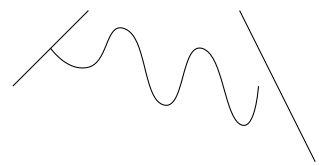
you could give a name to the arrows [*labels the arrows with* a, b, c, d], but that's practically never done, and then just write "$ba = dc$" or whatever it is, and prove it. But usually diagrams have more symbols than that. So I don't think the calculus of diagrams is on a very solid basis. Most of the things that people do is just claiming they are commutative. Well, Bourbaki has written some chapter on homological algebra where really there is no cheating. But it's difficult to read. The trouble is that very often if you wanted to give a name, you would end up by one string of letters equals another one. And then your only solution would be this follows from the definition. That's really one of the troubles of homological algebra.

What you find also in homological algebra. . . something about the spectral sequence of a map [*writes "The spectral sequence of* $X \to Y$*"*] (you have?) fibrant. This makes sense — "The" of course, "The" is the problem [*circles "The"*]. Because that makes sense only if you refer to a specific construction. There are various constructions in the literature, and it is not clear they give the same answer. You can see that there is a problem by asking yourself the following: Suppose you take two manifolds [*writes "* $U \times V$ *"*], you project them [*writes an arrow from* $U \times V$ *down to* U], two compact oriented manifolds, you choose orientations. In that case, on the E_2 term you have a well-defined invariant class (in top dimension?). Tensor product of here [*points to* U] and this [*points to* V]. So you end up with, whenever you have an orientation of U and V, you end up with an orientation of $U \times V$. But which one? Because there are two natural choices. So, for instance in Leray's spectral sequence, the basis comes first, so one writes the cohomology of the basis, cohomology of the fiber [*writes* $H^*(Y, H^*(B))$]. . . and one multiplies (?) product(s?) according to that. But Grothendieck, when he defines spectral sequences in his Tôhoku paper, just said there exists a spectral sequence. You have to look at the proof, and different people give different proofs. And I'm not sure— well, I'm particularly sure of the opposite— that they give the same answer. They give isomorphic terms, but not with the same. . . not with an obvious arrow.

So, I must say that I don't have much confidence in the science of occurring homological algebra at the moment. Okay, so that's homological algebra.

Topology

Topology! What are the troubles of topology? Well, they are essentially those of homological algebra, plus one. Plus: pictures. In the proof, you say, "Oh yes, we have this, see picture two." In picture two, you find something like this:



And if you ask the author "Was it important that it wiggles three times, or four times?" they'll go "No, of course not." "Was it important that they don't touch?" "Oh, it should touch!" I mean...

Well, there is a saying, says that a picture is worth— what is the standard? Is it a thousand, or twenty, or what?— thousand explanations. Yes, for the author, usually the author, a picture is very happy, says "Oh look, that is absolutely splendid, I understand everything." And for the reader, it takes a thousand explanations. That's topology.

Analysis

Analysis. Well, cheating in analysis is usually— especially in analytic number theory. The first trouble I want to point out is (for all analysis?). You have a theorem saying that [*writes "Thm. Let* $f\ldots,$*"*]... some complicated operator, depending on parameter [*writes "complicated oper. A"*]. Another theorem says that $|Af|$ is smaller than $C|f|$, where C is a constant. A beautiful word! You see, I don't think there is any place in mathematics, in Bourbaki or anything, that a constant is defined. What is meant is real number, strictly positive, and you call it a constant because it doesn't depend— so it's a real number not depending on *some* of the data. Now, very often the theorem is taken with "Let f." Now, when you introduce something by *Let*, exactly as if you took it, and you pin it to the blackboard with a nail [*mimes hammering a nail into the blackboard*]. Let! f is fixed. So that, *a priori*, in normal good mathematics the constant would depend on f. Of course not, also, doesn't want that. But they do not say. And the constant may depend, certainly on A, but on many other things. Another way of stating this which is more correct, which *is* correct, (especially is used?) is to use the Vinogradov symbol, that is, to write Af with the double [*writes* "$|Af| << |f|$"]. And then to put an index everything on which it depends. On A, or whatever, all parameters. So that's correct. On the other hand, what they do very often, is also to say, they have two functions $f(x), g(x)$, defined let's say for x strictly positive. Then we write $f(x) = O(g(x))$. Normally they should say, but they don't, for x going that direction [*writes "*$x \to +\infty$*"*].

But even that, that's alright. What does it mean exactly? It means there are two constants, not one, there are two constants x_0 and C, such that

$$|f(x)| \leq C|g(x)| \text{ for } x \geq x_0.$$

But very often, practically all the time, they will speak of *the* constant, implicit in O-notation. That makes sense only if you have made really clear what x_0 is. And in many instances in analytic number theory, it is not at all clear. You are computing, say, the prime function [*writes "$\tau(x)$"*], well, okay, you will assume something like that [*writes "$x \geq 2$"*]. But, in some theorems, like the Chebotarev Density Theorem, when you try to write explicit bounds, you find that x_0 is as difficult to choose as the constant. It's very important to find the x_0. So that's a typical way of cheating in analytic number theory which I think causes a lot of mistakes.

On "iff" (38:05)

Well maybe I should speak a little on the positive side. What can you do to escape those various things? I must have forgotten a lot. I made a list [*takes folded piece of paper out of pocket and checks it*…Oh yes, well, that's not very serious. It's about orthography. I wanted to comment a little bit on this [*writes "iff"*]. This is ugly English, but good mathematics. Because it is absolutely unambiguous: it means if and only if. So I cannot say much against it, except what you could do, for yourself, most of you work with a computer, to put on your computer an order saying every time I type i-f-f you print this [*underlines "if and only if"*]. This way you'd be happy, and I'd be happy to. I must say that the French are behaving as badly as the English [*writes "ssi"*]…*Si et seulement si.* Which fortunately, *l'Académie des Sciences* doesn't accept. If you send a compte rendu note, we will replace it with "si et seulement si." And I'm very glad to tell you that the Germans have never introduced [*writes "dannn"*]…*Dann urd nur dann.* So that doesn't exist [*crosses it out*]. I don't know Russian, so I can't tell you whether the Russians have invented something. So let's see, this was one of the things I forgot. Umm [*consults his list*]…

On Bourbaki Proofs (40:00)

Ah, yes. Ah yes, I want to say something on Bourbaki, yes. Because, we still have our Joan,[1] our author Joan, who wants to write as quickly as possible. I chose "Joan" because you could interpret it masculine or feminine [*writes "John, Joan"*]. I was going to give advice to Joan. In case the referee says "Oh my god, your proof really. . . your proof is not. . . not very good." Joan can always say "Oh yes, of course it's not a Bourbaki proof." You see, saying that it's not a Bourbaki proof usually makes people agree. I remember, once— (Bott's?) not here? No, he's not here. So, that was at (?) no, not at (?), at Bonn (?). There was a lecture by an old friend of mine on the work of a physicist. At the end of the one hour it was not clear to me whether he had proved something or not. So I asked him, "Is there a proof, or can one make this into a proof?" and he told me, "Oh, of course it's not a Bourbaki proof" and then everybody laughed. I was not really happy, but it made me think about what is the difference between a proof and a Bourbaki proof. So I can offer you the following: that a proof is something which is accepted by experts, and a Bourbaki proof is accepted by non-experts. And of course I favor the second choice.

On References and Proofs, again (42:18)

Now, to go back to how to try to write not too badly. There are some of the things I have said, some of the troubles, which are easy to escape. I mean, you can define your notation precisely, you can give references to books by giving the page number. I should still tell you something about references to books with page. I once refereed also, a paper on p-adic representations or something. Well where the author had a nice formula at one point and (he?) said "reference" and I looked at the reference and it was one of my books. Fortunately I knew essentially the formula in my book, so I knew it was not there. But, but, I first thought I would write and ask the proof, and then I made a counterexample. So that solves the equation, the formula was wrong.

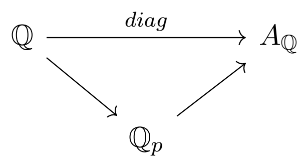
Some of the things I have said are easy to repair. Some are not. Those on diagrams commutative are serious. What I called cheating is practically impossible to escape. Every proof has some kind of cheating inside, and you cannot really give a totally explicit proof. So the question is, what do you leave out? And that's not

[1] *Transcriber's note: his accent makes this name sound like either "John" or "Joan."*

very clear. There are people who can do that, and others... I have another friend, fortunately I don't give names, he sent me about a year ago a proof he had made, he had solved some equation, a two-page proof. I thought about it. Then I gave it back to him, I said "The proof is wrong, but everything you have written is correct." Because, that's very often the case, people want to prove Theorem 1, we have this. Then they give an argument, then they stop. They stop, and they believe that with what they have proved, that implies the theorem, but the implication they did not detail. And that's where the mistake was. He had assumed something which is wrong. So very often, mistakes in proofs are in the non-written part, the implicit part.

On Identifications

Writing completely explicitly is certainly not possible. For instance, there is a (?) point of identifications. We have said all the time that we identify $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ and so on. But what does that mean? This is absolutely not true, I mean, $\mathbb{N}$ is not a part of $\mathbb{Z}$. If you follow— so, take the following set [*writes "3"*]. Everything is a set, particularly in the Zermelo-Fraenkel system, so 3 is a set. What is the cardinality of 3 [*writes "*card(3)*?"*]? I'm giving that in the Bourbaki system: $\mathbb{N}$ in Bourbaki, is representative of the finite sets. And the representative of the set with three elements is just 3. So in $\mathbb{N}$, cardinality of 3 is 3 [*writes "*$= 3,\ 3 \in \mathbb{N}$*" under* card(3)]. Elements of $\mathbb{Z}$ are pairs of integers with the same difference. So an element of $\mathbb{Z}$ has cardinality $\aleph_0$. So this is $\aleph_0$ when— I'm not very good at [*hesitates before writing* $\aleph_0$]— $\aleph_0$ if 3 is in $\mathbb{Z}$ or $\mathbb{Q}$ [*writes "*$= \aleph_0,\ 3 \in \mathbb{Z}, \mathbb{Q}$*" to the right of* $3 \in \mathbb{N}$]. But here [*points to* $\mathbb{R}$] you have Cauchy sequences, so that the cardinality is a power of the (quantity?) [*writes "*$= 2^{\aleph_0}$*"* to the right of $3 \in \mathbb{Z}, \mathbb{Q}$]. Okay, what does that mean? That means you should not ask the question. On the other hand, when we explain mathematics to other people, it is not quite compatible with what we say, to tell them "There are questions you should not ask." So, it's very disagreeable. You have to know, I took a rather trivial example, but there are much more amusing ones. If you take $\mathbb{Q}$ and the adeles of $\mathbb{Q}$, you have a diagonal embedding [*writes "*$\mathbb{Q} \xrightarrow{diag} A_{\mathbb{Q}}$"], but also you have the p-adics, which embed, and you have also embeddings.



So you have a whole collection of embeddings that are not compatible with each other. You have to know, which one you have in mind. Chevalley was a member of Bourbaki, and favored never to make an identification. But this is absolutely impossible, the textbook will be unreadable. You'd have to give a name to the injection of $\mathbb{N}$ into $\mathbb{Z}$ and so on. Here [*gestures to arrows in triangular diagram*] it might be good to have some name, but in that case [*gestures to* $\mathbb{N} \subset \mathbb{Z} \subset \ldots$] no.

So at the end, we have to admit that we can't write entirely rigorously, that it is a matter of art to know what to say and what not to say. Okay, I stop there.

Questions

I would be happy to have comments or questions, because one may have very different opinions on those things. Well, for instance, I didn't speak of that because it is a bit too difficult, but should one explain the ideas carefully, or give the full details, or both? My own feeling is that if you give me a proof with all the details, I can find the ideas behind, that's very difficult. If you give me the ideas and not the details, I may get completely stuck on some of the details. Also, I prefer a more detailed proof on paper. Because on paper, if you find it too long you just [*mimes flipping pages*] go to the next page. But if it is too short and if something is missing you may spend a lot of time trying to reconstruct it.

Okay, but I wanted other questions than the ones I ask myself. So... no? Oh, come on. You must know what is bad writing, I'm sure. Just read any text in the library. No? Yes?

(Inaudible)

I'm sure this happened with Bourbaki. Some member of Bourbaki would tell the other one "It's because you are stupid, and you have not found the correct statement which would give both of them." So normally, if that happens, it really should be— because that's, that's, I dislike that very much. "Oh yes, I've proved Theorem 1, to prove Theorem 2 is similar." (?) There is an expression in Latin to say that, you remember, no? Changing the things which should be changed. Oh, *mutatis mutandis*, yes. [*Tries to spell out a name, off screen. This goes on a while.*].

In plain English, it means "Changing what has to be changed." Yes, ah.

(Inaudible)

You mean in the paper? Yeah, if you can do that, it is always very useful. It belongs to the introduction, usually. Some people can do that very well. There is one way of writing papers where you have a very long introduction with all the statements and no proofs, and then the rest you give the proofs. That might be clearer, except that the notations should be clear for the theorems in the introduction. Very often you find in introduction Theorem 3.1, and so on, and the notation has not been introduced. But you are asking the serious question, how should one convey the ideas?

(Inaudible)

Motivation. . . I have a tendency to believe that the motivation is my own curiosité. I just want to know how these things work, and then I tell you how they work. Or I tell myself how they work. Real motivation. . . Well, I'm not sure I really believe in motivation. Something can look very strange at first and then it makes sense later. There are people who know how to write a very good introduction. Since Bott is not here, I can say his name. The topologists of my generation were quite good— Milnor writes beautifully and Atiyah. Some topics are less easy. I don't know how you can find the motivation. In the... well motivation, I mean it's a beautiful theorem: The theorem of Feit-Thompson: every finite group of odd order is solvable. The proof is 250 pages long or 300. Well, the theorem itself is the motivation, but the ideas of the proof to a non-specialist are invisible. Okay, anything else? Yes?

(Inaudible)

You think it was better when I was young? I don't think so [*laughs*]. I remember a paper in Topology in the Annals (?) they were not doing anything, they were making exact sequences going around, on groups of which they knew nothing. No, I don't want to blame it on your generation. But it is the natural tendency, the less effort, I think that's all. Maybe the computer is also for something because it's too easy now to take a piece you have written at some point and put it with another one and your notations don't agree. You can do many more things. When we had to type on paper and cut and glue, there was less possibility. I don't think the writing is worse now.

(Inaudible)

Oh? Well, okay, that's it. Again, thank you.

References

[1] Harvard's Basic Notions seminar. https://wstein.org/edu/basic/.

[2] J. Serre. How to write mathematics badly. Video: https://www.youtube.com/watch?v=ECQyFzzBHlo.